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STATIONARY SIMPLE WAVES ON A BAROTROPIC LIQUID SHEAR FLOW

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Stationary three-dimensional flows of a barotropic liquid in a gravity field are considered. In the shallow-water approximation, the Euler equations are transformed into a system of integrodifferential equations by the Euler–Lagrange change of coordinates. A system of simple-wave equations is obtained, for which the theorem of existence of a solution attached to a given shear flow is proved. As an example, a particular solution analogous to the solution of the problem of a gas flow around a convex angle is given.

Key words: *shallow water, simple waves, integrodifferential equations.*

1. System of Equations. We consider the Euler equations in the stationary case

$$(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}+\rho^{-1}\nabla p=\boldsymbol{g},\qquad \operatorname{div}(\rho\boldsymbol{u})=0,\tag{1.1}$$

where $\boldsymbol{u}=(u,v,w)^{\rm t}$ is the velocity vector, $\boldsymbol{g}=(0,0,-g)^{\rm t}$ is the acceleration of gravity, ρ is the density [$\rho=R(p)$ is a prescribed function], and p is the pressure. The spatial variables are $x,y\in(-\infty,+\infty)$ and $z\in(0,h(x,y))$. System (1.1) for stationary flows of a barotropic liquid in a gravity field over a flat bottom with a free boundary is supplemented by kinematic boundary conditions at the bottom and at the free boundary and by a dynamic condition at the free boundary:

$$w|_{z=0}=0,\qquad w|_{z=h(x,y)}=uh_x+vh_y,\qquad p|_{z=h(x,y)}=p_a$$

(p_a is the atmospheric pressure).

Stationary simple waves traveling in a constant-density liquid were studied by Teshukov [1]. The plane-parallel case of simple-wave propagation in a barotropic liquid layer was examined by Elemesova [2].

After the substitution of variables

$$x=L_0x',\quad y=L_0y',\quad z=H_0z',\quad u=\sqrt{gH_0}\,u',\quad v=\sqrt{gH_0}\,v',\quad w=\sqrt{gH_0}\,H_0w'/L_0,$$

$$\rho=R_0\rho',\qquad p=R_0gH_0p',$$

where L_0 and H_0 are the characteristic horizontal and vertical scales and R_0 has the dimensions of density, Eqs. (1.1) in dimensionless variables acquire the following form (the primes are omitted):

$$uu_x+vu_y+wu_z+\rho^{-1}p_x=0,\qquad uv_x+vv_y+wv_z+\rho^{-1}p_y=0,$$

$$\varepsilon^2(uw_x+vw_y+ww_z)+\rho^{-1}p_z=-1,\qquad \operatorname{div}(\rho\boldsymbol{u})=0$$

($\varepsilon=H_0/L_0$). In the shallow-water approximation, the parameter ε is assumed to be small compared to unity. In the limit $\varepsilon\to 0$, the third equation of momenta has the form of the hydrostatic-pressure law

$$p_z=-\rho.\tag{1.2}$$

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From here, we obtain the formula for pressure

$$\int\limits_{p_a}^{p(x,y,z)} \frac{dp'}{R(p')} = h(x,y) - z,$$

which yields the equalities $p_x = \rho h_x$ and $p_y = \rho h_y$.

The remaining equations of system (1.1) can be simplified by passing into the Euler–Lagrange coordinate system [3]

$$x = x', \qquad y = y', \qquad z = \Phi(x', y', \lambda),$$

where the function Φ is the solution of the boundary-value problem

$$\begin{gathered} u(x,y,\Phi)\,\Phi_x + v(x,y,\Phi)\,\Phi_y = w(x,y,\Phi), \\ \Phi|_{\lambda=0} = 0, \qquad \Phi|_{\lambda=1} = h. \end{gathered} \tag{1.3}$$

After this change of variables, the Lagrangian coordinate $\lambda \in [0,1]$ enumerates the material surfaces of the liquid from $z = 0$ ($\lambda = 0$) to $z = h$ ($\lambda = 1$).

The first two momentum equations and the continuity equation acquire the form

$$uu_x + vu_y + h_x = 0, \qquad uv_x + vv_y + h_y = 0,$$

$$(u\rho\Phi_\lambda)_x + (v\rho\Phi_\lambda)_y = 0.$$

We introduce a new unknown function $H(x,y,\lambda) = \rho\Phi_\lambda$. Suppose that the Jacobian of the change of variables Φ_λ is greater than 0, then $H > 0$.

Since

$$h = \int\limits_0^1 \Phi_\lambda\,d\lambda = \int\limits_0^1 \rho^{-1} H\,d\lambda,$$

then the last system of equations can be transformed into the system [4]

$$(\boldsymbol{u}\cdot\nabla)\,\boldsymbol{u} + \tau \int\limits_0^1 \nabla H\,d\lambda = 0, \qquad \operatorname{div}(H\boldsymbol{u}) = 0. \tag{1.4}$$

Here $\boldsymbol{u}$ is the projection of the velocity vector onto the horizontal plane: $\boldsymbol{u} = (u,v)^{\mathrm{t}}$; the operators ∇ and div act through the variables x and y, and $\tau = \tau(x,y)$ is the specific volume at the bottom:

$$\tau(x,y) = (\rho(x,y,0))^{-1} = \Big(R\Big(p_a + \int\limits_0^1 H\,d\lambda\Big)\Big)^{-1}.$$

From the found solution of system (1.4), $u(x,y,\lambda)$, $v(x,y,\lambda)$, and $H(x,y,\lambda)$, the initial variables can be reconstructed as follows. From Eq. (1.2), it follows that

$$p_\lambda = -H, \tag{1.5}$$

whence, we obtain

$$p(x,y,\lambda) = p_a + \int\limits_\lambda^1 H(x,y,\nu)\,d\nu. \tag{1.6}$$

Next, since $\Phi_\lambda = H/\rho = -p_\lambda/R(p)$, the dependence $\lambda(x,y,z)$ (1.6) can be implicitly obtained from the found pressure (1.6) by the formula

$$z = \Phi(x,y,\lambda) = \int\limits_{p(x,y,\lambda)}^{p(x,y,0)} \frac{dp'}{R(p')}. \tag{1.7}$$

The vertical coordinate of the velocity w is given by the first equation in (1.3).

2. Simple Waves. We will call solutions of system (1.4) of the form

$$\boldsymbol{u} = \boldsymbol{u}(\alpha(x, y), \lambda), \qquad H = H(\alpha(x, y), \lambda) \tag{2.1}$$

simple waves [1]. These solutions correspond to the initial-system solutions of the form $\boldsymbol{u} = \boldsymbol{u}(\alpha(x, y), z)$ and $p = p(\alpha(x, y), z)$.

After substitution of (2.1) into Eqs. (1.4), we obtain the system

$$(\boldsymbol{u} \cdot \nabla\alpha)\boldsymbol{u}_\alpha + \tau \int\limits_0^1 H_\alpha \, d\lambda \, \nabla\alpha = 0, \qquad (\boldsymbol{u} \cdot \nabla\alpha)H_\alpha + H(\boldsymbol{u}_\alpha \cdot \nabla\alpha) = 0 \tag{2.2}$$

for the functions $\boldsymbol{u}(\alpha, \lambda)$, $H(\alpha, \lambda)$, and $\alpha(x, y)$.

We introduce an auxiliary function $\boldsymbol{n} = |\nabla\alpha|^{-1}\nabla\alpha$ being the normal to the contour lines of the simple wave. Then, Eqs. (2.2) transform into

$$\boldsymbol{u}_\alpha = -\frac{\tau}{u_n} \int\limits_0^1 H_\alpha \, d\lambda \, \boldsymbol{n}; \tag{2.3}$$

$$H_\alpha = \frac{H\tau}{u_n^2} \int\limits_0^1 H_\alpha \, d\lambda, \tag{2.4}$$

where $u_n = \boldsymbol{u} \cdot \boldsymbol{n}$.

It follows from Eq. (2.4) that the normal $\boldsymbol{n}$ satisfies the equation

$$\tau \int\limits_0^1 \frac{H}{u_n^2} \, d\lambda = 1. \tag{2.5}$$

Thus, we have system (2.3)–(2.5) for the functions $\boldsymbol{u}(\alpha, \lambda)$ and $H(\alpha, \lambda)$. The simple-wave parameter $\alpha(x, y)$ is defined as follows. On the line $\alpha(x, y) = \text{const}$, the following equality holds:

$$\nabla\alpha \cdot d\boldsymbol{x} = 0 \qquad \text{or} \qquad \boldsymbol{n} \cdot d\boldsymbol{x} = 0.$$

It follows from Eq. (2.5) that $\boldsymbol{n} = \boldsymbol{n}(\alpha)$, and the last equation can be integrated:

$$\boldsymbol{n}(\alpha) \cdot \boldsymbol{x} = C(\alpha) \tag{2.6}$$

[$C(\alpha)$ is an arbitrary function]. If $\boldsymbol{n}'(\alpha) \cdot \boldsymbol{x} - C'(\alpha) \neq 0$, then, by the theorem of the implicit function, Eq. (2.6) allows us to determine the local values of the function $\alpha(x, y)$. Equation (2.6) means that the simple-wave contour lines are, in fact, straight lines.

For further studies, we choose the function $\alpha = \int\limits_0^1 H \, d\lambda$ as the simple-wave parameter.

We make the polar change of variables $u = q\cos\theta$ and $v = q\sin\theta$ in the plane (u, v) (q and θ are new sought functions of α and λ). We represent the direction $\boldsymbol{n}$ in the form $\boldsymbol{n} = (-\sin\gamma, \cos\gamma)$, where $\gamma = \gamma(\alpha)$, the angle between the straight line $\alpha = \text{const}$ and the x axis, is the sought function. Then, Eqs. (2.3)–(2.5) become

$$q_\alpha = -\tau/q; \tag{2.7}$$

$$\theta_\alpha = -\tau\cot(\theta - \gamma)/q^2; \tag{2.8}$$

$$H_\alpha = H\tau/(q^2\sin^2(\theta - \gamma)); \tag{2.9}$$

$$\tau \int\limits_0^1 \frac{H \, d\lambda}{q^2\sin^2(\theta - \gamma)} = 1. \tag{2.10}$$

We consider Eq. (2.10) in more detail. Note that the function

$$\chi(\gamma) = 1 - \tau \int \frac{H}{q^2\sin^2(\theta - \gamma)} \, d\lambda$$

is periodic with a period π. In addition, the solutions of the equation $\chi(\gamma) = 0$, which differ from one another by a multiple of π, characterize the same line $\alpha = \text{const}$. For this reason, it suffices to consider the equation $\chi(\gamma) = 0$ over one period.

Let $\theta(\lambda) \in [\theta^0, \theta^1]$ for $\lambda \in [0, 1]$ and, simultaneously, $\theta^0 = \theta|_{\lambda=0}$, $\theta^1 = \theta|_{\lambda=1}$, and $\theta_\lambda > 0$. We consider the case $\theta^1 - \theta^0 < \pi$ because, otherwise, the function $\chi(\gamma)$ is undeterminate on the real axis. We put $\theta^2 = \theta^0 + \pi$ and consider $\chi(\gamma)$ for $\gamma \in (\theta^1, \theta^2)$.

We calculate $\chi''(\gamma)$:

$$\chi''(\gamma) = -2\tau \int_0^1 \frac{H}{q^2} \, \frac{1 + 2\cos^2(\theta - \gamma)}{\sin^4(\theta - \gamma)} \, d\lambda < 0.$$

It follows from the last inequality that the function $\chi(\gamma)$ is convex upward and, since $\chi \to -\infty$ as $\gamma \to \theta^1 + 0$ and $\gamma \to \theta^2 - 0$, the function $\chi(\gamma)$ has a single maximum $\gamma_* \in (\theta^1, \theta^2)$. Therefore, for $\chi(\gamma_*) > 0$, Eq. (2.10) has two roots $\gamma_{1,2}$ in the segment (θ^1, θ^2), which correspond to two families of simple waves.

3. Simple Waves on Flows without Velocity Shear. Integration of Eqs. (2.7)–(2.10) is possible in the case of a flow without velocity shear along the vertical line, i.e., for $u_\lambda = v_\lambda = 0$ or $q = q(\alpha)$ and $\theta = \theta(\alpha)$. Here, the velocity field is the same as in the simple wave in a steady isentropic two-dimensional gas flow (Prandtl–Mayer flow) [5]:

$$\theta = \theta_0 \pm \mu(q), \qquad \mu(q) = \int \sqrt{\mathrm{M}^2 - 1} \, \frac{dq}{q} \tag{3.1}$$

(M is the Mach number). Indeed, we find from Eq. (2.10) that

$$\gamma_{1,2} = \theta \pm \arcsin(\sqrt{\tau\alpha}/q).$$

Then, according to (2.8), the flow-velocity direction is given by the formula

$$\theta(\alpha) = \theta_0 \pm \int_{\alpha_0}^{\alpha} \frac{\tau(\alpha')}{q^2(\alpha')} \Big(\frac{q^2(\alpha')}{\tau(\alpha')\alpha'} - 1 \Big)^{1/2} d\alpha'.$$

After the substitution $\alpha = \alpha(q)$, the integral, by virtue of (2.7), acquires the form

$$\int_{q_0}^{q} \sqrt{\frac{q^2}{\tau\alpha} - 1} \, \frac{dq}{q}.$$

If we take the function $\bar\rho = \alpha$ as the density in the flow under study and the function $\bar p = \int \tau(\alpha)\alpha \, d\alpha$ as the pressure, then the speed of sound becomes $\bar c^2 = d\bar p / d\bar\rho = \tau\alpha$ and the Mach number $\mathrm{M} = q/\sqrt{\tau\alpha}$. As a result, we obtain formula (3.1). Recall that $\alpha = \int_0^1 H \, d\lambda$ and, by definition of the function H, it follows that $\bar\rho = \int_0^h \rho \, dz$ is the mass of the liquid column from the bottom to the free surface.

To completely determine the solution, we also have to find the pressure. Integrating Eqs. (2.9), we obtain

$$H(\alpha, \lambda) = D(\lambda)\alpha,$$

where $D(\lambda)$ is an arbitrary function. Then, it follows from (1.6) that

$$p(\alpha, \lambda) = p_a + \alpha \int_\lambda^1 D(\lambda') \, d\lambda'.$$

The dependence on the z coordinate can be established from Eq. (1.7).

4. Existence of Simple Waves on a Shear Flow. The shear flow is understood as particular solutions of system (1.4) of the form [1]

$$\boldsymbol{u} = \boldsymbol{u}(\lambda), \qquad H = H(\lambda),$$

which correspond to the solutions $\boldsymbol{u} = (u(z), v(z), 0)^{\mathrm{t}}$ and $p = p(z)$ of the initial system.

The problem of attachment of a simple wave to a specified shear flow $\boldsymbol{u}_0(\lambda)$ and $H_0(\lambda)$ is formulated as follows. One has to solve system (2.7)–(2.10) with the initial conditions

$$q|_{\alpha=\alpha_0} = q_0(\lambda), \qquad \theta|_{\alpha=\alpha_0} = \theta_0(\lambda), \qquad H|_{\alpha=\alpha_0} = H_0(\lambda), \tag{4.1}$$

where $\boldsymbol{u}_0 = q_0(\cos\theta_0, \sin\theta_0)$.

To prove the existence of a solution of system (2.7)–(2.10), we may conveniently use, instead of the final relation (2.10), the differential equation

$$\gamma_\alpha = \frac{\tau}{2}\Big(\int\limits_0^1 \frac{H\cos(\theta-\gamma)}{q^2\sin^3(\theta-\gamma)}\,d\lambda\Big)^{-1}\Big(R'\Big(p_a + \int\limits_0^1 H\,d\lambda\Big)\int\limits_0^1 \frac{H\,d\lambda}{q^2\sin^2(\theta-\gamma)} - 3\int\limits_0^1 \frac{H\,d\lambda}{q^4\sin^4(\theta-\gamma)}\Big), \tag{4.2}$$

obtained from (2.10) by differentiation with respect to α. The initial condition

$$\gamma|_{\alpha=\alpha_0} = \gamma_0 \tag{4.3}$$

can be found from the equation

$$\Big(R\Big(p_a + \int\limits_0^1 H_0\,d\lambda\Big)\Big)^{-1}\int\limits_0^1 \frac{H_0\,d\lambda}{q_0^2\sin^2(\theta_0-\gamma_0)} = 1.$$

System (2.7)–(2.9) and (4.2) with the initial conditions (4.1) and (4.3) can be represented as a Cauchy problem for the differential equation in the space of functions λ:

$$\frac{d\boldsymbol{V}}{d\alpha} = \boldsymbol{F}(\boldsymbol{V}), \qquad \boldsymbol{V}(\alpha_0) = \boldsymbol{V}_0. \tag{4.4}$$

Here $\boldsymbol{V} = (q(\lambda), \theta(\lambda), H(\lambda), \gamma)$ and $\boldsymbol{V}_0 = (q_0(\lambda), \theta_0(\lambda), H_0(\lambda), \gamma_0)$.

To prove the existence of the solution of problem (4.4), we use the known theorem of the theory of differential equations in Banach spaces [6]: if, for $\|\boldsymbol{V} - \boldsymbol{V}_0\| \leqslant \eta$, the function $\boldsymbol{F}$ satisfies the conditions

$$\|\boldsymbol{F}(\boldsymbol{V})\| \leqslant M_1; \tag{4.5}$$

$$\|\boldsymbol{F}(\boldsymbol{V}_2) - \boldsymbol{F}(\boldsymbol{V}_1)\| \leqslant M_2\|\boldsymbol{V}_2 - \boldsymbol{V}_1\|, \tag{4.6}$$

then there exists a number $\delta > 0$ ($\delta = \min\{\eta/M_1, 1/M_2\}$) such that problem (4.4) has a unique solution in the interval $(\alpha_0 - \delta, \alpha_0 + \delta)$.

We introduce a norm in the space of vector-functions $\boldsymbol{V}(\lambda)$ as follows:

$$\|\boldsymbol{V}\| = \max_\lambda q(\lambda) + \max_\lambda |\theta(\lambda)| + \max_\lambda H(\lambda) + |\gamma|.$$

Let $\eta > 0$ be such that the initial data $\boldsymbol{V}_0$ satisfy the conditions $q_0(\lambda) > 2\eta$, $H_0(\lambda) > 2\eta$, $\chi(\gamma_*(\boldsymbol{V}_0)) > 0$, $2\eta < |\theta_0(\lambda) - \gamma_0| < \pi - 2\eta$, and $|\gamma_*(\boldsymbol{V}_0) - \gamma_0| > 2\eta$. Then, in the ball $\|\boldsymbol{V} - \boldsymbol{V}_0\| < \eta$, the following inequalities are valid:

$$q(\lambda) > \eta, \quad H(\lambda) > \eta, \quad \eta < |\theta(\lambda) - \gamma| < \pi - \eta, \quad |\gamma_*(\boldsymbol{V}) - \gamma| > \epsilon(\eta) \tag{4.7}$$

[$\epsilon(\eta) > 0$]. Inequalities (4.7) enable obtaining estimates (4.5) and (4.6) for system (2.7)–(2.9), (4.2) with certain constants $M_1(\eta)$ and $M_2(\eta)$. Inequality (4.5) is valid by virtue of continuity of the right side of $\boldsymbol{F}(\boldsymbol{V})$ in the ball $\|\boldsymbol{V} - \boldsymbol{V}_0\| < \eta$ and, to obtain inequality (4.6), the Gâteaux derivative $\boldsymbol{F}'(\boldsymbol{V})$ in the ball $\|\boldsymbol{V} - \boldsymbol{V}_0\| < \eta$ should be estimated.

5. Example. Below, we give a particular solution of system (2.7)–(2.10), analogous to the solution of the problem of the flow around a convex corner in gasdynamics (the Prandtl–Mayer flow).

Note that Eq. (2.7) yields the Bernoulli-integral analog

$$q^2(\alpha, \lambda) + 2\int\limits_{p_a}^{p_a+\alpha} \frac{dp}{R(p)} = q_m^2(\lambda), \tag{5.1}$$

where $q_m(\lambda)$ is an arbitrary positive function.

We seek particular solutions in which the absolute velocity q is independent of λ. This means that $q_m = \text{const}$ in integral (5.1).

It follows from Eqs. (2.8) and (2.9) that

$$\theta_\lambda = A(\lambda)H, \tag{5.2}$$

where $A(\lambda)$ is an arbitrary function. Consider the particular solution with $A = \text{const}$. Equation (2.10) acquires the form

$$ARq^2 = \cot(\theta^0 - \gamma) - \cot(\theta^1 - \gamma). \tag{5.3}$$

It follows from Eq. (5.2) that $\theta^1(\alpha) - \theta^0(\alpha) = A\alpha$. Using the formula for the cotangent of a difference, we can transform equality (5.3) into a quadratic equation for $\cot(\theta^1 - \gamma)$; solving the resultant quadratic equation, we obtain

$$\gamma_{1,2}(\alpha) = \theta^1 + \operatorname{arccot}\Big(\frac{ARq^2}{2} \pm \sqrt{\Big(\frac{ARq^2}{2}\Big)^2 + ARq^2\cot(A\alpha) - 1}\Big). \tag{5.4}$$

Formula (5.4) gives different real roots $\gamma_{1,2}$ if the radicand in the argument of the arctangent is positive. The latter condition is equivalent to the condition $\chi(\gamma_*) > 0$:

$$ARq^2 > 2\tan(A\alpha/2).$$

Substituting expression (5.4) for γ into Eq. (2.8) with $\lambda = 1$, we obtain θ^1:

$$\theta^1(\alpha) = \theta^1_0 + \int\limits_{\alpha_0}^{\alpha}\Big(\frac{A}{2} \pm \sqrt{\frac{A^2}{4} + \frac{A\cot(A\alpha)}{Rq^2} - \frac{1}{R^2q^4}}\Big)\,d\alpha$$

(θ^1_0 is an arbitrary constant).

It follows from Eqs. (5.2) and (1.5) that

$$\theta(\alpha,\lambda) = \theta^1(\alpha) - A(p(\alpha,\lambda) - p_a).$$

Note that the dependence of pressure $p(\alpha,\lambda)$ on the Euler coordinate z can be found from the following equation resulting from (1.2):

$$z = \int\limits_{p(\alpha,\lambda)}^{p(\alpha,0)}\frac{dp'}{R(p')}.$$

Here

$$p(\alpha,0) = p_a + \int\limits_0^1 H\,d\lambda = p_a + \alpha.$$

Hence, the Euler flow pattern will be completely determined if we find the simple-wave parameter $\alpha(x,y)$. To completely determine the flow in the Euler–Lagrange coordinates x, y, λ, we also have to find the function $H(x,y,\lambda)$.

Consider a particular solution with $C(\alpha) \equiv 0$ in Eq. (2.6), which corresponds to a simple wave centered at the origin. Then, the simple-wave parameter α can be found from the equation

$$\tan\gamma(\alpha) = y/x.$$

In the polar coordinates $x = r\cos\varphi$ and $y = r\sin\varphi$, this equation means that $\gamma(\alpha) = \varphi$. Then the function $\alpha(\varphi)$ can be found from the equation

$$\varphi = \theta^1(\alpha) + \operatorname{arccot}\Big(\frac{AR(\alpha)q^2(\alpha)}{2} \pm \sqrt{\Big(\frac{AR(\alpha)q^2(\alpha)}{2}\Big)^2 + AR(\alpha)q^2(\alpha)\cot(A\alpha) - 1}\Big).$$

The values of A, q_m, θ^1_0, and α_0 can be calculated from the initial data:

$$\theta^1_0 = \theta_0(1), \quad \alpha_0 = \int\limits_0^1 H_0\,d\lambda, \quad A = \frac{\theta_0(1) - \theta_0(0)}{\alpha_0}, \quad q_m = \sqrt{q_0^2 + 2\int\limits_{p_a}^{p_a+\alpha_0}\frac{dp}{R(p)}}.$$

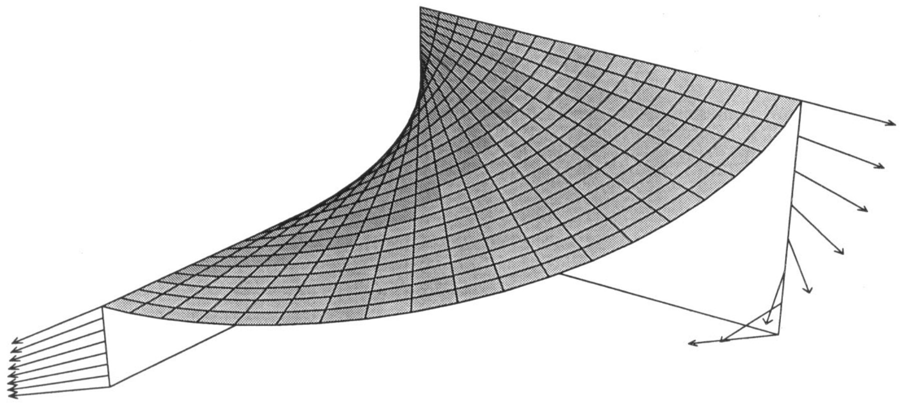

Fig. 1

The particular solution obtained, considered over the interval $[\alpha_0, \alpha_1]$, matches two steady shear flows of depths

$$h_0 = \int\limits_{p_a}^{p_a+\alpha_0} \frac{dp}{R(p)}, \qquad h_1 = \int\limits_{p_a}^{p_a+\alpha_1} \frac{dp}{R(p)}$$

with velocities $(q_0, \theta_0(\lambda))$ and $(q(\alpha_1), \theta(\alpha_1, \lambda))$. The region occupied by the simple wave is bounded by the rays $\varphi(\alpha_0)$ and $\varphi(\alpha_1)$.

Figure 1 shows the shape of the free surface and the velocity field at the beginning and at the end of the interval $[\alpha_0, \alpha_1]$ for the polytropic dependence $p = S\rho^{æ}$ ($S = \text{const}$, $æ > 1$).

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